Short Papers

Graph Design of p-i-n Diode Phase Shifters

KENZO WATANABE, MEMBER, IEEE, MASAKI ARIMA, AND TATSUO YAMAMOTO, MEMBER, IEEE

Abstract—A synthesis procedure of the impedance-transforming network in a p-i-n diode phase shifter is given. A representation of a reflection performance on the impedance plane is used successfully to determine the impedance matrix of the network. The procedure is straightforward and its validity is demonstrated by a prototype 90° phase shifter at 10 GHz.

I. Introduction

The p-i-n diode phase shifter is now an indispensable component in PSK communication systems and phased array antennas. Of the various types of phase shifter proposed so far [1], [2], a reflection-type circuit employing a hybrid coupler is most common because of low cost, small size, and easy integration [3], [4]. The reflection performance required for this circuit is constant signal amplitude with a specified phase difference between the binary bias states of the diode. The diode cannot usually meet this requirement by itself and the tuning elements or impedance-transforming networks are inevitable in front of the diode.

The design problem of the impedance-transforming network was first attacked by Starski [5]. His method, however, requires a laborious calculation in addition to the precise knowledge of diode parameters. Recently Atwater has proposed a novel method which has allowed the simple design by making use of a bilinear transformation [6].

The design method described in this paper is based on the bilinear relation between the reflection coefficient and the diode impedance. This relation is visualized on the impedance plane to graphically find the matching impedance. This process gives a clear insight into the physical aspects of the problem. The matching impedance is then related to the impedance matrix in terms of which the impedance-transforming network is synthesized. A design example is given to demonstrate the procedure and its validity.

II. Design Theory

A. Driving Point Impedance

The impedance-transforming network is characterized in terms of the impedance matrix as shown in Fig. 1. The driving-point impedance of the network when terminated by a resistor of 1 Ω is given by

$$Z_{\text{in}}(s)=Z_{11}(s)\frac{\Delta Z(s)/Z_{11}(s)+1}{Z_{22}(s)+1} \tag{1}$$

Manuscript received January 5, 1981; revised February 16, 1981. This work was supported by Amano Institute of Industrial Technology, Japan.

K. Watanabe and T. Yamamoto are with the Research Institute of Electronics, Shizuoka University, Hamamatsu 432, Japan.

M. Arima was with the Research Institute of Electronics, Shizuoka University, Japan. He is now with Matsushita Electric Industrial Company, Ltd., Kadoma 571, Japan.

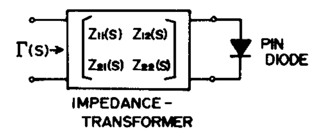

Fig. 1. A reflection-type phase shifter consisting of an impedance-transformer and a p-i-n diode.

where

$$\Delta Z(s)=Z_{11}(s)Z_{22}(s)-Z_{12}^{2}(s) \tag{2}$$

and s denotes a complex frequency.

This impedance should be a positive real function because the network is composed of real components. Expressing it as

$$Z_{\text{in}}(s)=\frac{m_1(s)+n_1(s)}{m_2(s)+n_2(s)}=\frac{m_1(s)}{n_2(s)}\,\frac{n_1(s)/m_1(s)+1}{m_2(s)/n_2(s)+1} \tag{3}$$

and comparing (3) with (1), we have

$$Z_{11}(s)=\frac{m_1(s)}{n_2(s)} \qquad Z_{22}(s)=\frac{m_2(s)}{n_2(s)}$$

$$Z_{12}(s)=\frac{\sqrt{m_1(s)m_2(s)-n_1(s)n_2(s)}}{n_2(s)} \tag{4}$$

where $m_1(s)$ and $m_2(s)$ represent even polynomials of s, and $n_1(s)$ and $n_2(s)$, odd polynomials.

The set of $Z_{11}(s)$, $Z_{12}(s)$, and $Z_{22}(s)$ are always compatible in representing a two-port network. The synthesis procedure using the relation in (4) is called Darlington's driving-point impedance method [7].

B. Reflection Coefficient

The driving-point impedance of the network when terminated with a p-i-n diode in place of 1-Ω resistor is given by (1) with the diode impedance $Z_d(s)$ replacing 1. The reflection coefficient is then expressed as

$$\Gamma(s)=u(s)\frac{Z_d(s)-Z_m(s)}{Z_d(s)+Z_m(-s)} \tag{5}$$

where

$$u(s)=\frac{m_1(s)-n_2(s)}{m_1(s)+n_2(s)} \tag{6}$$

$$Z_m(s)=\frac{m_2(s)-n_1(s)}{m_1(s)-n_2(s)}. \tag{7}$$

$Z_m(s)$ is the matching impedance introduced by Atwater [6].

On the $j\omega$-axis, $|u(s)|=1$ and $Z_m(-s)=Z_m^*(s)$. The requirement for the reflection coefficient is then formulated as follows:

$$e^{j\phi}\frac{Z_d^1(j\omega)-Z_m(j\omega)}{Z_d^1(j\omega)+Z_m^*(j\omega)}=\frac{Z_d^2(j\omega)-Z_m(j\omega)}{Z_d^2(j\omega)+Z_m^*(j\omega)} \tag{8}$$

where Z_d^1 and Z_d^2 represent the diode impedance in its "ON" and "OFF" states, and ϕ is a specified phase difference.

0018-9480/81/0800-0829$00.75 ©1981 IEEE

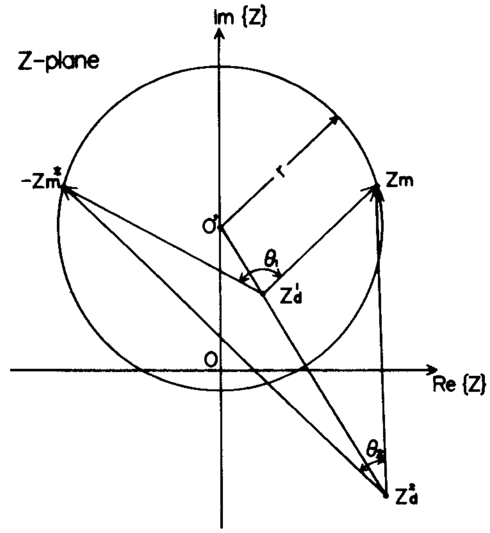

Fig. 2. Apollonius' circle with limiting points Z_d^1 and Z_d^2 centered on the imaginary axis. The radius is given by

$$r=\sqrt{\overline{0'Z_d^1}\cdot\overline{0'Z_d^2}}$$

Z_m and $-Z_m^*$ are symmetrical about the imaginary axis.

It should be noted here that the matching impedance Z_m is interpreted as the virtual source impedance which matches the reflection coefficient of the diode to the required one. The problem of the reflection match is then reduced to that of the impedance match between the virtual and practical source impedances.

C. Graphical Determination of Matching Impedance

It is a tedious task to find the matching impedance by calculation for a given set of Z_d^1 and Z_d^2. The graphic method, instead, which makes use of the bilinear relation in (8) visualized on the impedance plane is effective to find Z_m easily.

Rearranging (8) as

$$\frac{|Z_m(j\omega)-Z_d^1(j\omega)|}{|Z_m(j\omega)-Z_d^2(j\omega)|}=\frac{|-Z_m^*(j\omega)-Z_d^1(j\omega)|}{|-Z_m^*(j\omega)-Z_d^2(j\omega)|}=\text{const.} \quad (9)$$

one notices that Z_m and $-Z_m^*$ should be located at positions whose distances from Z_d^1 and Z_d^2 have a constant ratio. The loci of these points constitute the family of coaxial circles known as the "circles of Apollonius" [8]. The condition that Z_m and $-Z_m^*$ are symmetric about the imaginary axis requires for the Apollonius' circle to be centered on the imaginary axis. The constant amplitude condition of the reflection coefficient thus specifies the unique circle. The procedure is illustrated in Fig. 2; plotting Z_d^1 and Z_d^2 on the impedance plane, finding the intersection 0′ of the straight line through Z_d^1 and Z_d^2 and the imaginary axis, and then drawing the circle centered on 0′ with a radius of geometric mean of $|0'Z_d^1|$ and $|0'Z_d^2|$, we have the required circle.

To determine Z_m and $-Z_m^*$ on this circle the phase condition is used. The phasors in Fig. 2 explain the phase relation; θ_1 is the reflection phase in Z_d^1 state and θ_2 is in Z_d^2 state. It is clear by inspection that there exists always one and only one pair of points that meets the phase specification $\phi=\theta_1-\theta_2$. These points are Z_m and $-Z_m^*$, the point in the right-hand plane corresponding to Z_m because the numerator in (8) is smaller than the denominator, i.e., $|\Gamma(s)|\leq 1$.

Once the matching impedances Z_m's are determined at several frequencies of interest, the synthesis of the network is a routine task. Generating an appropriate rational function for $Z_m(s)$ and decomposing it into the even and odd polynomials as in (7) give the impedance matrix entries in (4). An example will be given in the next section.

III. Design Example

The simplest rational function is a bilinear function

$$Z_m(s)=R_m(s)+jX_m(s)=\frac{1-as}{1-bs}. \quad (10)$$

Then, by (4), we have

$$Z=\frac{1}{bs}\begin{pmatrix} 1 & \sqrt{1-abs^2} \\ \sqrt{1-abs^2} & 1 \end{pmatrix}. \quad (11)$$

If one regards s in (11) as Richard's variable [9]

$$s=j\tan\beta l=j\tan\theta_m \quad (12)$$

then (11) represents the impedance matrix of a transmission line with a characteristic impedance $1/b=Z_{0m}$ and electrical length $\theta_m\cdot Z_{0m}$ and θ_m can be found by (10)

$$Z_{0m}=\sqrt{\frac{R_m^2+X_m^2-R_m}{R_m-1}} \quad (13)$$

$$\theta_m=\tan^{-1}\frac{1-R_m}{X_m}Z_{0m}. \quad (14)$$

If R_m and X_m are denormalized, (13) and (14) become the same as (16) and (17) in [6], respectively.

A 90° phase shifter at 10 GHz has been implemented to confirm the above synthesis procedure. The diode was so mounted as to terminate a 50-Ω microstrip line formed on an alumina substrate 0.64 mm thick having a relative dielectric constant of 10.3. The normalized diode impedance was $0.09+j1.04$ when forward-biased with a current of 10 mA, and $0.16-j1.31$ when reverse-biased with 10 V. Using these values the matching impedance was found graphically to be

$$Z_m=2.49+j0.89.$$

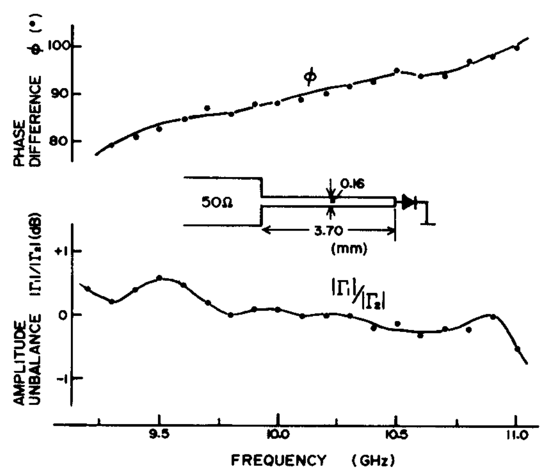

Fig. 3. The reflection performance of a prototype 90° phase shifter.

Z_{0m} and θ_m of the impedance-transforming network were obtained to be 1.74 and 109°, respectively.

The measured reflection performance is shown in Fig. 3 together with the strip pattern designed using the dispersive model of microstrip line. The phase error and the amplitude unbalance at 10 GHz are 2° and 0.1 dB, respectively. These are considered due to the fringing parasitics at the impedance step.

A mechanization of a matching network by means of one section of microstrip line, as in this example, is not always possible because the attainable characteristic impedance is limited. In those cases, a stub or short length of transmission line should be incorporated in front of the diode to transform the diode impedances to new values which allow a practical design of a microstrip matching section.

IV. Conclusions

A synthesis method for an impedance-transforming network of a p-i-n diode phase shifter has been presented. It features the simplified design made possible by the graphical determination of the matching impedance. A design example which assumes the matching impedance of the form of bilinear function has been given to demonstrate the procedure.

The synthesis of a broad-band network is also possible if a matching impedance of higher order polynomials is assumed and their coefficients are determined at several frequencies of interest. Since the coefficients determined graphically are only of first-order approximation, a computer-aided optimization would be effective to a final design.

Acknowledgment

The authors wish to thank K. Kondo for his help in the fabrication and the measurement of the prototype phase shifter.

References

[1] R. W. Burns, R. L. Holden, and R. Tang, "Low cost design technique for semiconductor phase shifters," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-22, pp. 675–687, June 1974.

[2] R. V. Garver, "Broadband diode phase shifters," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-20, pp. 314–323, May 1972.

[3] J. F White, "Diode phase shifters for array antennas," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-22, pp. 658–674, June 1974.

[4] F. G. Terrio, R. J Stockton, and W D. Sato, "A low cost P-I-N diode phase shifters for airborne phased-array antennas," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-22, pp. 688–692, June 1974.

[5] J. P. Starski, "Optimization of the matching network for a hybrid coupler phase shifter," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-25, pp. 662–666, Aug. 1977.

[6] H. A. Atwater, "Reflection coefficient transformation for phase-shift circuits," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-28, pp. 563–568, June 1980.

[7] M E. Van Valkenburg, *Introduction to Modern Network Synthesis.* New York: Wiley, 1960, ch. 14, p. 402.

[8] L. V. Ahlfors, *Complex Analysis.* New York: McGraw-Hill, 1966, ch. 3, p. 84

[9] G. C. Temes and J. W. LaPatra, *Circuit Synthesis and Design.* New York: McGraw-Hill, 1977, ch. 8, p 329.